27. $\dfrac{x\cos\frac{\pi}{4}(2\sin x - x\cos x)}{\sqrt{2}\sin^2 x}$

28. $\dfrac{1+\tan x - x\sec^2 x}{(1+\tan x)^2}$

29. $(x+\sec x)\left(1-\sec^2 x\right)+(x-\tan x).(1+\sec x\tan x)$

30. $\dfrac{\sin x - n\,x\cos x}{\sin^{n+1}x}$

EXERCISE 14.1

1. (i) This sentence is always false because the maximum number of days in a month is 31. Therefore, it is a statement.

(ii) This is not a statement because for some people mathematics can be easy and for some others it can be difficult.

(iii) This sentence is always true because the sum is 12 and it is greater than 10. Therefore, it is a statement.

(iv) This sentence is sometimes true and sometimes not true. For example the square of 2 is even number and the square of 3 is an odd number. Therefore, it is not a statement.

(v) This sentence is sometimes true and sometimes false. For example, squares and rhombus have equal length whereas rectangles and trapezium have unequal length. Therefore, it is not a statement.

(vi) It is an order and therefore, is not a statement.

(vii) This sentence is false as the product is (–8). Therefore, it is a statement.

(viii) This sentence is always true and therefore, it is a statement.

(ix) It is not clear from the context which day is referred and therefore, it is not a statement.

(x) This is a true statement because all real numbers can be written in the form $a + i \times 0$.

2. The three examples can be:

(i) Everyone in this room is bold. This is not a statement because from the context it is not clear which room is referred here and the term bold is not precisely defined.

(ii) She is an engineering student. This is also not a statement because who 'she' is.

(iii) "$\cos^2\theta$ is always greater than 1/2". Unless, we know what θ is, we cannot say whether the sentence is true or not.

EXERCISES 14.2

1. (i) Chennai is not the capital of Tamil Nadu.
   (ii) $\sqrt{2}$ is a complex number.
   (iii) All triangles are equilateral triangles.
   (iv) The number 2 is not greater than 7.
   (v) Every natural number is not an integer.
2. (i) The negation of the first statement is "the number x is a rational number." which is the same as the second statement" This is because when a number is not irrational, it is a rational. Therefore, the given pairs are negations of each other.
   (ii) The negation of the first statement is "x is an irrational number" which is the same as the second statement. Therefore, the pairs are negations of each other.
3. (i) Number 3 is prime; number 3 is odd (True).
   (ii) All integers are positive; all integers are negative (False).
   (iii) 100 is divisible by 3,100 is divisible by 11 and 100 is divisible by 5 (False).

EXERCISE 14.3

1. (i) "And". The component statements are:
   All rational numbers are real.
   All real numbers are not complex.
   (ii) "Or". The component statements are:
   Square of an integer is positive.
   Square of an integer is negative.
   (iii) "And". the component statements are:
   The sand heats up quickly in the sun.
   The sand does not cool down fast at night.
   (iv) "And". The component statements are:
   $x = 2$ is a root of the equation $3x^2 - x - 10 = 0$
   $x = 3$ is a root of the equation $3x^2 - x - 10 = 0$
2. (i) "There exists". The negation is
   There does not exist a number which is equal to its square.
   (ii) "For every". The negation is
   There exists a real number x such that x is not less than $x + 1$.
   (iii) "There exists". The negation is
   There exists a state in India which does not have a capital.

3. No. The negation of the statement in (i) is "There exists real number x and y for which $x + y \neq y + x$", instead of the statement given in (ii).

4. (i) Exclusive

(ii) Inclusive

(iii) Exclusive

EXERCISE 14.4

1. (i) A natural number is odd implies that its square is odd.

(ii) A natural number is odd only if its square is odd.

(iii) For a natural number to be odd it is necessary that its square is odd.

(iv) For the square of a natural number to be odd, it is sufficient that the number is odd

(v) If the square of a natural number is not odd, then the natural number is not odd.

2. (i) The contrapositive is

If a number x is not odd, then x is not a prime number.

The converse is

If a number x in odd, then it is a prime number.

(ii) The contrapositive is

If two lines intersect in the same plane, then they are not parallel

The converse is

If two lines do not interesect in the same plane, then they are parallel

(iii) The contrapositive is

If something is not at low temperature, then it is not cold

The converse is

If something is at low temperature, then it is cold

(iv) The contrapositive is

If you know how to reason deductively, then you can comprehend geometry.

The converse is

If you do not know how to reason deductively, then you can not comprehend geometry.

(v) This statement can be written as "If x is an even number, then x is divisible by 4".

The contrapositive is, If x is not divisible by 4, then x is not an even number.

The converse is, If x is divisible by 4, then x is an even number.

3. (i) If you get a job, then your credentials are good.

(ii) If the banana tree stays warm for a month, then it will bloom.

(iii) If diagonals of a quadrilateral bisect each other, then it is a parallelogram.

(iv) If you get A^+ in the class, then you do all the exercises in the book.

4. **a** (i) Contrapositive

(ii) Converse

b (i) Contrapositive

(ii) Converse

EXERCISE 14.5

5. (i) False. By definition of the chord, it should intersect the circle in two points.

(ii) False. This can be shown by giving a counter example. A chord which is not a dimaeter gives the counter example.

(iii) True. In the equation of an ellipse if we put $a = b$, then it is a circle (Direct Method)

(iv) True, by the rule of inequality

(v) False. Since 11 is a prime number, therefore $\sqrt{11}$ is irrational.

Miscellaneous Exercise on Chapter 14

1. (i) There exists a positive real number x such that $x-1$ is not positive.

(ii) There exists a cat which does not scratch.

(iii) There exists a real number x such that neither $x > 1$ nor $x < 1$.

(iv) There does not exist a number x such that $0 < x < 1$.

2. (i) The statement can be written as "If a positive integer is prime, then it has no divisors other than 1 and itself.

The converse of the statement is

If a positive integer has no divisors other than 1 and itself, then it is a prime.

The contrapositive of the statement is

If positive integer has divisors other than 1 and itself then it is not prime.

(ii) The given statement can be written as "If it is a sunny day, then I go to a beach.

The converse of the statement is

If I go to beach, then it is a sunny day.

The contrapositive is

If I do not go to a beach, then it is not a sunny day.

(iii) The converse is

If you feel thirsty, then it is hot outside.

The contrapositive is

If you do not feel thirsty, then it is not hot outside.